# Two-Port to Three-Port Noise-Wave Transformation for CAD Applications

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*Abstract*— **A two-port-to-three-port noise wave transformation is presented that complements the similar transformation for $S$-parameters. Others have shown that transistor two-port $S$-parameters can be converted to three-port $S$-parameters when the common terminal is used as the third port. This paper shows how to calculate the three-port noise waves and noise temperatures from the two-port noise waves and temperatures. The transformation considers the general case where the common terminal of the three-port is not perfectly grounded. The formulation is then extended to the general case of a $(n-1)$-port-to-$n$-port transformation.**

## I. Introduction

The two-port $S$-parameters of a three terminal device, like a transistor, with one terminal grounded can be converted to three-port $S$-parameters when the grounded terminal is used as a third terminal [1], [2]. A complimentary noise wave transformation would be useful to evaluate transistor circuits of arbitrary topology. Authors have tabulated how the noise figure parameters of a device with one grounded terminal and input terminal are transformed to any other grounded and input terminal [3], [4]. Also, others have shown how noise figure is effected by series-feedback, parallel-feedback and cascade networks [4], [5].

This paper shows a two-port-to-three-port noise transformation technique that can be used to model three terminal devices for CAD applications. Noise waves were introduced as an alternative model for the noise properties of two-port networks [6]–[11]. This has been shown to apply to networks of three or more ports and arbitrary topologies [12], [13]. It has not been shown how the characteristic minimum noise figure, $F_{\text{min}}$, optimum noise match, $\Gamma_{\text{opt}}$, and noise resistance, $R_n$, for a transistor two-port can be transformed to a three-port noise model when the grounded, or common terminal, is used as the third port. Most CAD programs will find the three-port Y-parameter or $S$-parameters of a transistor from the two-port $S$-parameters in a data file. This paper will show how the characteristic noise parameters found in the transistor files or data sheets can be converted to three-port noise-wave temperatures. Then, the transistor can be used in any arbitrary topology when these three-port noise characteristics are known.

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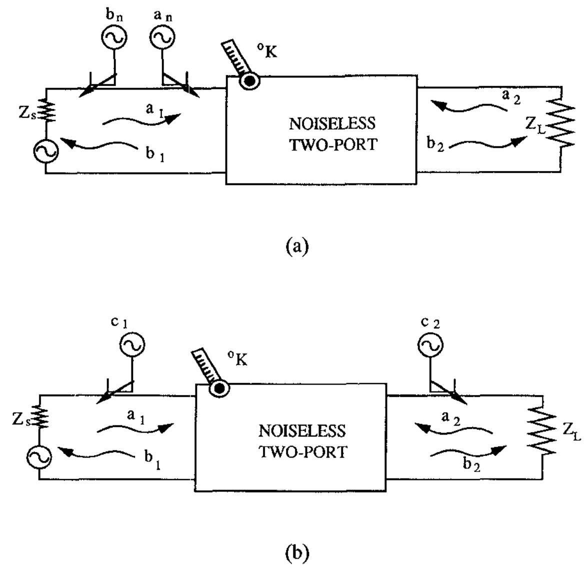


Fig. 1. Modeling of a noisy two-port with a noiseless two-port and two noise waves, (a) input referenced noise waves $a_n$ and $b_n$, and (b) two outgoing noise waves $c_1$ and $c_2$.

## II. Two-Port Noise Temperatures

Noise waves are defined much like the incident and reflected power waves in transmission line theory. Consider the circuit in Fig. 1(a) of a noiseless two-port and two noise wave sources $a_n$ and $b_n$. These noise waves are included in describing the two-port linear network by the equation

$$\begin{pmatrix} b_1 - b_n \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} a_1 + a_n \\ a_2 \end{pmatrix}. \tag{1}$$

The amplitude and phase of these noise waves are not known, but their auto-correlation and cross-correlation can be found from the noise temperatures

$$C_n = \begin{pmatrix} \left\langle |a_n|^2 \right\rangle & \langle a_n^* b_n \rangle \\ \langle a_n b_n^* \rangle & \left\langle |b_n|^2 \right\rangle \end{pmatrix} = k\,\Delta f \begin{pmatrix} T_a & T_c e^{j\phi_c} \\ T_c e^{-j\phi_c} & T_b \end{pmatrix}, \tag{2}$$

where the brackets $\langle\ \rangle$ indicate the time average of the quantity inside, $k$ is Boltzman's constant, and $\Delta f$ is the noise bandwidth (usually one Hz). These temperatures are related to $F_{\text{min}}$, $\Gamma_{\text{opt}}$, and $R_n$ by the equations listed in Appendix A [12].

Fig. 1(b) shows another noise model for the noisy two-port. The noise generated by the two-port is modeled as two outgoing waves $c_1$ and $c_2$. Again, these noise waves are only known by their auto-correlation and cross-correlation temperatures. The two-port linear matrix equation describing the noisy two-port is

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}. \tag{3}$$

The noise-wave correlation matrix describing the circuit in Fig. 1(b) can be expressed as a function of the original input referenced noise waves, Fig. 1(a):

$$\begin{aligned} \boldsymbol{C}_{2p} &= \begin{pmatrix} \langle |c_1|^2 \rangle & \langle c_1 c_2^* \rangle \\ \langle c_1^* c_2 \rangle & \langle |c_2|^2 \rangle \end{pmatrix} \\ &= k\,\Delta f \begin{pmatrix} T_1 & T_3 \\ T_3^* & T_2 \end{pmatrix} = \boldsymbol{T}\boldsymbol{C}_n\boldsymbol{T}\dagger \end{aligned} \tag{4}$$

where the dagger indicates the Hermitian conjugate and

$$\boldsymbol{T} = \begin{pmatrix} S_{11} & 1 \\ S_{21} & 0 \end{pmatrix}.$$

Or, the elements of $\boldsymbol{C}_{2p}$ can be calculated directly from $F_{\min}$, $\Gamma_{\text{opt}}$, and $R_n$ [15].

## III. Two-Port-to-Three-Port Transformation

Suppose there were originally three noise waves, one for each terminal, before terminal three was grounded as shown in Fig. 2. The two noise waves in (3) are a function of the three-port noise waves and the reflection coefficient of the termination on the third terminal. Much like $S$-parameters, the noise properties of the transistor characterized by three-port noise waves are uniquely defined by two-port noise wave temperatures when one terminal is grounded. The transistor is characterized by the linear matrix equation

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} + \begin{pmatrix} c_1' \\ c_2' \\ c_3' \end{pmatrix}, \tag{5}$$

where the three-port $S$-parameters are used. The two-port $S$-parameters and noise figure parameters in the device file correspond to the situation where terminal three, usually the source or emitter, is grounded. Let us consider the general case when terminal three is terminated with an arbitrary impedance $Z_3$ as shown in Fig. 3. This causes a reflection coefficient of $\Gamma_3$ to be seen by the common terminal of the device (when terminal three were perfectly grounded, $\Gamma_3 = -1$). The termination $Z_3$ at a noise temperature of $T_s$ introduces an additional noise wave $a_{n3}$ incident on port 3. The new two-port circuit is characterized by the set of linear equations

$$\begin{aligned} b_1 = {} & a_1\left(S_{11} + \frac{\Gamma_3 S_{13} S_{31}}{1 - \Gamma_3 S_{33}}\right) \\ & + a_2\left(S_{12} + \frac{\Gamma_3 S_{13} S_{32}}{1 - \Gamma_3 S_{33}}\right) + c_1, \end{aligned} \tag{6}$$

$$\begin{aligned} b_2 = {} & a_1\left(S_{21} + \frac{\Gamma_3 S_{23} S_{31}}{1 - \Gamma_3 S_{33}}\right) \\ & + a_2\left(S_{12} + \frac{\Gamma_3 S_{23} S_{32}}{1 - \Gamma_3 S_{33}}\right) + c_2, \end{aligned} \tag{7}$$

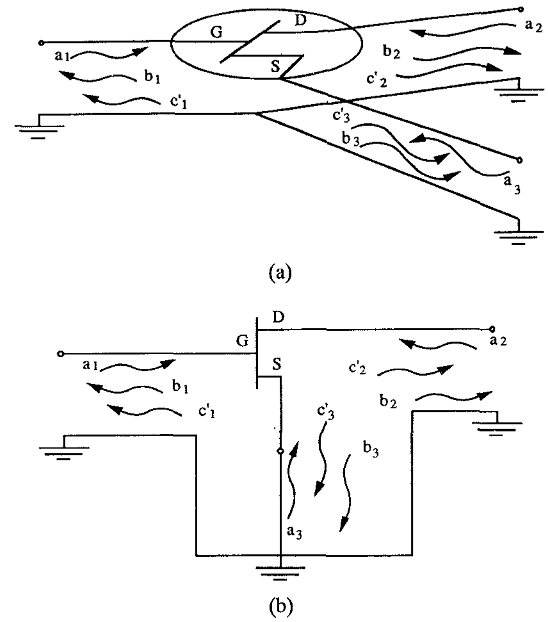


Fig. 2. A noisy transistor three-port (a) with three outgoing noise waves, and (b) with terminal three grounded, reflecting $b_3$ and $c_3$ back as $-a_3$.

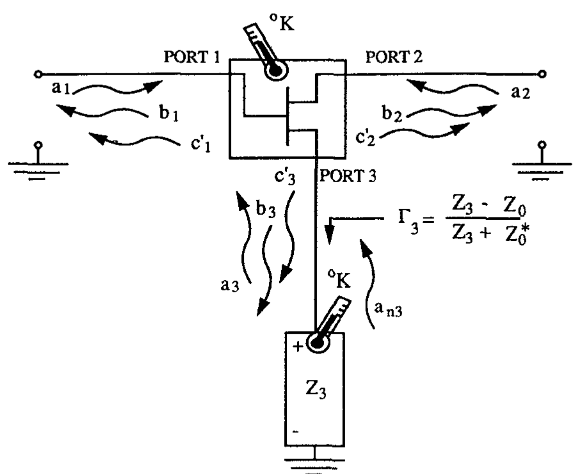


Fig. 3. A general three port with a termination of $\Gamma_3$ on port 3 giving rise to an additional nose wave $a_{n3}$ due to the temperature of $Z_3$.

where the two-port $S$-parameters are given by the quantities in the parentheses, and the two-port noise waves, $c_1$ and $c_2$, are given by

$$c_1 = c_3' \frac{\Gamma_3 S_{13}}{1 - \Gamma_3 S_{33}} + a_{n3} \frac{S_{13}}{1 - \Gamma_3 S_{33}} + c_1', \tag{8}$$

$$c_2 = c_3' \frac{\Gamma_3 S_{23}}{1 - \Gamma_3 S_{33}} + a_{n3} \frac{S_{23}}{1 - \Gamma_3 S_{33}} + c_2'. \tag{9}$$

where

$$\langle |a_{n3}|^2 \rangle = T_s k\,\Delta f \left| 1 - |\Gamma_3|^2 \right|. \tag{10}$$

Equations (6) and (7) show how two-port $S$-parameters and noise figure parameters are effected by a series element $Z_3$ imbedded in the three-port common terminal (bond wires or VIAs for example). For the removal or extraction of $Z_3$ from data files or measurements, a two-port to three port $S$-parameter and noise wave conversion method is needed.

The three-port $S$-parameters are found by using the principle that

$$\sum_{i=1}^{3} S_{ij} = \sum_{j=1}^{3} S_{ij} = 1 \tag{11}$$

for indefinite networks [1]. Manipulating the $S$-parameters in (6) and (7) and using the principle (11), the three-port $S$-parameters can be found that remove the embedded series element $Z_3$ from the common terminal:

$$S_{33} = \frac{\sum_{i=1}^{2}\sum_{j=1}^{2} S_{ij}^{o} - \Gamma_3 - 1}{1 - 3\Gamma_3 - \sum_{i=1}^{2}\sum_{j=1}^{2} S_{ij}^{o}}, \tag{12}$$

$$S_{i3} = \left(\frac{1-\Gamma_3 S_{33}}{1-\Gamma_3}\right)\left(1 - \sum_{j=1}^{2} S_{ij}^{o}\right), \quad (i = 1,2), \tag{13}$$

$$S_{mj} = \left(\frac{1-\Gamma_3 S_{33}}{1-\Gamma_3}\right)\left(1 - \sum_{i=1}^{2} S_{ij}^{o}\right), \quad (i = 1,2), \tag{14}$$

$$S_{ij} = S_{ij}^{o} - \left(\frac{\Gamma_3 S_{i3} S_{3j}}{1-\Gamma_3 S_{33}}\right), \quad (i,j = 1,2), \tag{15}$$

where $S_{ij}^{o}$ $(i,j = 1,2)$ are the original two-port $S$-parameters and $\Gamma_3$ is the reflection coefficient of the termination on port 3.

Equations (8) and (9) show two equations for $c_1$ and $c_2$ and three unknown three-port noise waves $c_1'$, $c_2'$, and $c_3'$. The three-port noise waves can be calculated by finding a third independent equation. The elements of the three-port indefinite correlation matrix satisfy the principle that

$$\sum_{i=1}^{3} \langle c_i' c_j'^{*} \rangle = \sum_{j=1}^{3} \langle c_i' c_j'^{*} \rangle = 0. \tag{16}$$

Since the noise waves will eventually be time averaged, we can say that

$$c_1' + c_2' + c_3' = 0 \tag{17}$$

even though (17) does not have to be true for the instantaneous quantities. With (17) and the previous two equations (8) and (9), the three-port noise waves are found from the two-port noise waves:

$$c_1' = c_1\left(1 + \frac{\Gamma_3 S_{13}}{1-\Gamma_3}\right) + c_2\left(\frac{\Gamma_3 S_{13}}{1-\Gamma_3}\right) + a_{n3}\left(\frac{S_{13}}{\Gamma_3 - 1}\right), \tag{18}$$

$$c_2' = c_1\left(\frac{\Gamma_3 S_{23}}{1-\Gamma_3}\right) + c_2\left(1 + \frac{\Gamma_3 S_{23}}{1-\Gamma_3}\right) + a_{n3}\left(\frac{S_{23}}{\Gamma_3 - 1}\right), \tag{19}$$

$$c_3' = (c_1 + c_2)\left(\frac{1-\Gamma_3 S_{33}}{\Gamma_3 - 1}\right) - a_{n3}\left(\frac{1 - S_{33}}{\Gamma_3 - 1}\right). \tag{20}$$

Equations (18) through (20) will remove the series-feedback $\Gamma_3$ on the common terminal. When the common terminal is assumed to be perfectly grounded, $\Gamma_3$ should be set equal to $-1$.

The three-port noise correlation matrix $C'$ can be found from the two-port noise correlation by

$$C_{3p} = \left|\frac{1}{1-\Gamma_3}\right|^2 \left(KC_{2p}K\dagger - T_s k\,\Delta f \left|1 - |\Gamma_3|^2\right| DD\dagger\right) \tag{21}$$

where

$$K = \begin{pmatrix} 1 + \Gamma_3(S_{13} - 1) & \Gamma_3 S_{13} \\ \Gamma_3 S_{23} & 1 + \Gamma_3(S_{23} - 1) \\ 1 - \Gamma_3 S_{33} & 1 - \Gamma_3 S_{33} \end{pmatrix},$$

$$D = \begin{pmatrix} S_{13} \\ S_{23} \\ S_{33} - 1 \end{pmatrix}$$

and $K$ and $D$ use the three-port $S$-parameters found using (12) through (15). The three-port noise correlation matrix define the three-port noise waves, and the transistor can be connected in any arbitrary network [13], [14].

The inverse transformation can be easily derived by using equations (8) and (9). Since the removal of a parasitic $\Gamma_3$ has been carried throughout the two-port-to-three-port formulation above, a new series-feedback element $\Gamma_3$ can be added to the three-port device:

$$C_{2p} = K' C_{3p} K'\dagger + T_s k\,\Delta f \frac{\left|1 - |\Gamma_3|^2\right|}{|1 - \Gamma_3 S_{33}|^2} D' D'\dagger, \tag{22}$$

where

$$K' = \begin{pmatrix} 1 & 0 & \frac{\Gamma_3 S_{13}}{1-\Gamma_3 S_{33}} \\ 0 & 1 & \frac{\Gamma_3 S_{23}}{1-\Gamma_3 S_{33}} \end{pmatrix} \quad \text{and} \quad D' = \begin{pmatrix} S_{13} \\ S_{23} \end{pmatrix}$$

The following example illustrates the use of (21) and (22) for reconfiguring a transistor and adding series feedback.

## IV. Example

Suppose that a 24 GHz cascode amplifier is to be designed with a noise figure of 2.25 or less. Several transistors are available for this experiment. For this example, we want to know the NE 20200 chip transistor could be used for this application. From the NEC data sheets, the common-source output-drain $S$-parameters at 24 GHz are

$$S_{11}^{o} = 0.67\,\angle -149, \quad S_{12}^{o} = 0.10\,\angle 16, \\ S_{21}^{o} = 1.85\,\angle 39, \quad S_{11}^{o} = 0.52\,\angle 96,$$

and the noise figure parameters are

$$F_{\min} = 2.03, \quad r_n = 0.27, \quad \Gamma_{\text{opt}} = 0.42\,\angle 148,$$

for the bias point of two volts drain to source and 10 mA drain current. The first stage could be properly matched to give a 2.03 noise figure. If the noise figure contribution of the second common-gate stage is less than 0.22, then this transistor can be used for both stages of the cascode amplifier. The data sheets don't give the common-gate noise parameters for this transistor. They can be measured, consuming several man-hours, or they can be calculated using noise-wave temperatures.

The three input reference noise temperatures are found to be

$$T_a = 417.76, \quad T_b = 376.24, \quad T_c e^{i\phi_c} = 283.48\angle 32.00.$$

The two-port noise temperatures are

$$T_1 = 183.98, \quad T_2 = 1429.78, \quad T_3 = 11.25\,\angle 46.46.$$

The two-port $S$-parameters and noise data include the contribution of the source bond wires. These wires will be removed from the data when calculating the three-port parameters and temperatures. The three-port $S$-parameters are calculated with a $\Gamma_3 = .9\,\angle 165$ so as to remove the embedded source ground wires below. The three-port noise temperatures are calculated as shown below. To use equation (22), elements of the three-port $S$-parameter and noise correlation matrix are rearranged so that the source is port 1 and the gate is port 3. For this example, it is assumed that the gate has a poor ground connection with a reflection coefficient of $\Gamma_3 = 0.8\,\angle 150$. The new two-port noise temperatures are

$$T_1 = 671.8, \quad T_2 = 7582, \quad T_3 = 1859\,\angle -161.6.$$

The new input referenced noise temperature are found by inverting (4):

$$T_a = 789.6, \quad T_b = 318.7, \quad T_c e^{i\phi_c} = 284.9\,\angle 14.43$$

Finally, the new noise figure parameters for the common-gate input-source is

$$F_{\min} = 3.451, \quad r_n = .48, \quad \Gamma_{\text{opt}} = 0.277\,\angle 165.6$$

Since the first stage can only have a gain of 7, the common gate stage could add more than .6 to the noise figure of the cascode.

## V. Conclusions

The new transformations presented here have been checked against previous methods by calculating numeric examples. Dahlke's results [4] were used to convert the transistor noise parameters to another common terminal and input terminal configuration. Then Albinsson's [6] formulation was used to add series feedback to the transistor. The resultant values for $F_{\min}$, $\Gamma_{\text{opt}}$, and $R_n$ were the same as those calculated by the noise temperature method presented here.

The noise temperature two-port and three-port transformation has advantages over previous methods. The transistor can then be connected in any arbitrary network using the existing data files. Then techniques developed by others [10], [13], [14] can be used to evaluate the noise properties of networks. This method uses only $S$-parameters and noise temperatures. It is not required to convert to $Y$ and/or $Z$ parameters to change the common terminal or add series-feedback. Appendix B gives the general $(n-1)$-port to $n$-port transformations. Also, series-feedback elements at a temperature other than 290° K and with negative resistance can be easily added to the transistor.

## Appendix A

The equations to convert from minimum noise figure, $F_{\min}$, optimum noise source reflection coefficient, $\Gamma_{\text{opt}}$, and noise resistance, $R_n$, to the three input referenced noise temperatures when $\Delta f = 1$ Hz are

$$T_a = (F_{\min} - 1)T_o + \frac{4R_n T_o |\Gamma_{\text{opt}}|^2}{|1 + \Gamma_{\text{opt}}|^2}, \tag{30}$$

$$T_b = \frac{4R_n T_o}{|1 + \Gamma_{\text{opt}}|^2} - (F_{\min} - 1)T_o, \tag{31}$$

$$T_c = \frac{4R_n T_o |\Gamma_{\text{opt}}|}{|1 + \Gamma_{\text{opt}}|^2}, \tag{32}$$

$$\phi_c = \pi - \angle \Gamma_{\text{opt}}, \tag{33}$$

where $T_o$ is the standard temperature of 290° K. The inverse relations are

$$\angle \Gamma_{\text{opt}} = \pi - \phi_c, \tag{34}$$

$$|\Gamma_{\text{opt}}| = \frac{T_a + T_b}{2|T_c|} - \sqrt{\left(\frac{T_a + T_b}{2|T_c|}\right)^2 - 1}, \tag{35}$$

$$R_n = Z_o \frac{|T_c||1 + \Gamma_{\text{opt}}|^2}{|\Gamma_{\text{opt}}| 4T_o}, \tag{36}$$

$$F_{\min} = 1 + \frac{T_a - T_b}{2T_o} - \frac{|T_c|\left(|\Gamma_o|^2 - 1\right)}{2T_o|\Gamma_o|}. \tag{37}$$

$$\begin{aligned}
&S_{11} = 0.822\,\angle -20.6, && S_{12} = 0.603\,\angle 36.9, && S_{13} = 0.9994\,\angle 21.7,\\
&S_{21} = 1.755\,\angle 44.5, && S_{22} = 0.774\,\angle -94.6, && S_{23} = 0.494\,\angle -111.0,\\
&S_{31} = 0.546\,\angle 70.9, && S_{32} = 0.707\,\angle 36.7, && S_{33} = 0.270\,\angle 27.9,
\end{aligned}$$

$$\begin{aligned}
&T_{11} = 357.17, && T_{12} = 752.29\,\angle -174.73, && T_{13} = 396.39\,\angle 8.62,\\
&T_{21} = 752.29\,\angle 174.73, && T_{22} = 1841.78, && T_{23} = 1089.96\,\angle -177.87,\\
&T_{31} = 391.36\,\angle -17.03, && T_{32} = 1089.96\,\angle 77.87, && T_{33} = 694.34,
\end{aligned}$$

$$K = \begin{pmatrix} 1+\Gamma_m(S_{1m}-1) & \Gamma_m S_{1m} & \cdots & \Gamma_m S_{1m} \\ \Gamma_m S_{2m} & 1+\Gamma_m(S_{1m}-1) & \cdots & \Gamma_m S_{2m} \\ \cdot & \cdot & \cdot & \cdot \\ \Gamma_m S_{(m-1)m} & \Gamma_m S_{(m-1)m} & \cdots & 1+\Gamma_m(S_{(m-1)m}-1) \\ 1-\Gamma_m S_{mm} & 1-\Gamma_m S_{mm} & \cdots & 1-\Gamma_m S_{mm} \end{pmatrix}$$

## Appendix B

This analysis can be extended to any number of ports. Suppose there is a $(m-1)$-port with a grounded terminal or ground reference internal to the $(m-1)$-port. When this internal ground reference is disconnected and used as the $m$th port, the $S$-parameters and noise temperatures can be converted to a $m$-port. A series element on the $m$th terminal can also be removed. The new $S$-parameters of the $m$-port are

$$S_{mm} = \frac{2-\Gamma_m - m + \sum_{i=1}^{m-1}\sum_{j=1}^{m-1} S_{ij}^o}{1-m\Gamma_m - \sum_{i=1}^{m-1}\sum_{j=1}^{m-1} S_{ij}^o}, \quad (23)$$

$$S_{im} = \left(\frac{1-\Gamma_m S_{mm}}{1-\Gamma_m}\right)\left(1-\sum_{j=1}^{m-1} S_{ij}^o\right), \quad i=1,2,\cdots,m-1, \quad (24)$$

$$S_{mj} = \left(\frac{1-\Gamma_m S_{mm}}{1-\Gamma_m}\right)\left(1-\sum_{i=1}^{m-1} S_{ij}^o\right), \quad j=1,2,\cdots,m-1, \quad (25)$$

$$S_{ij} = S_{ij}^o - \left(\frac{\Gamma_m S_{im} S_{mj}}{1-\Gamma_m S_{mm}}\right), \quad i,j=1,2,\cdots,m-1, \quad (26)$$

using the original $(m-1)$-port $S$-parameters, $S_{ij}^o$ $(i,j = 1,2,\cdots,m-1)$. The $m$-port noise correlation is

$$C_m = \left|\frac{1}{1-\Gamma_m}\right|^2 \left(KC_{(m-1)}K\dagger - T_s k\,\Delta f\left|1-|\Gamma_m|^2\right|DD\dagger\right) \quad (27)$$

where there is $K$, which is enumerated at the top of this page. And

$$D = \begin{pmatrix} S_{1m} \\ S_{2m} \\ \cdot \\ \cdot \\ S_{(m-1)m} \\ S_{mm}-1 \end{pmatrix}.$$

Extending (22) gives the $m$-port-to-$(m-1)$-port transformation by connecting a termination with a reflection coefficient of $\Gamma_m$ to the $m$th port. The new $(m-1)$-port $S$-parameters are

$$S'_{ij} = S_{ij} + \left(\frac{\Gamma_m S_{im} S_{mj}}{1-\Gamma_m S_{mm}}\right), \quad i,j=1,2,\cdots,m-1, \quad (28)$$

the new $(m-1)$-port noise wave correlation matrix is

$$C_{(m-1)} = K'C_m K'\dagger + T_s k\,\Delta f \frac{\left|1-|\Gamma_m|^2\right|}{|1-\Gamma_m S_{mm}|^2} D'D'\dagger, \quad (29)$$

where

$$K' = \begin{pmatrix} 1 & 0 & \cdots & 0 & \frac{\Gamma_m S_{1m}}{1-\Gamma_m S_{mm}} \\ 0 & 1 & \cdots & 0 & \frac{\Gamma_m S_{2m}}{1-\Gamma_m S_{mm}} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdots & 1 & \frac{\Gamma_m S_{(m-1)m}}{1-\Gamma_m S_{mm}} \end{pmatrix} \text{ and}$$

$$D' = \begin{pmatrix} S_{1m} \\ S_{2m} \\ \cdot \\ \cdot \\ \cdot \\ S_{(m-1)m} \end{pmatrix}.$$

These noise temperature equations can be used to analyze the noise temperature of transistor reflection gain amplifiers (one-port equations) and multiple gate transistors.

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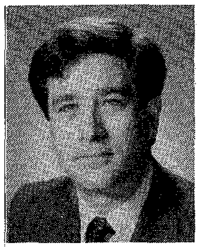

**Lynn A. Carpenter** (M'72) was born in Cushing, OK on April 25, 1943. He received his B.S. degree in physics from Oklahoma State University in 1964 and his M.S. and Ph.D. in physics from the University of Illinois at Urbana Champaign in 1966 and 1971. He is an Associate Professor of Electrical Engineering at the Pennsylvania State University, University Park, PA.

His research interest in microwaves and radar systems includes work on MMIC design and test and CAD applications of amplifiers, mixers and oscillators. He recently was on sabbatical at the Applied Physics Laboratory, Johns Hopkins University in the Space Department working on MMIC and dielectric resonator oscillators. He has help develop several CAD programs such as RESOMICS by Murata Erie, State College and CASCADE by Amplifonix.